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Bayesian Probabilistic Finite Element Model Updating of the UCF (University of Central Florida) Benchmark Structure

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Abstract—*Finite Element (FE) model updating has gained much importance in recent time due to its contribution in predicting the behavior of physical system with high precision monitoring structural damages etc. In the present work, FE model updating of the University of Florida (UCF) benchmark structure, which is a scale bridge model, is carried out using Bayesian probabilistic approach. At first, a FE model of the bridge is created both using SAP-2000 and MATLAB-2013 based on the given geometry and material information. Subsequently the analytical natural frequencies and mode shapes are obtained. Then the model updating of the structure is performed using the incomplete modal data obtained from dynamic testing of the laboratory model. The Bayesian Probability applied in this paper uses a sequence of identified modal parameter data sets to find the most probable model as well as system modal frequencies and the full system mode shapes. It is seen that there is a good agreement between analytical and experimental results after FE model updating.*

1. INTRODUCTION

Finite element method is a popular way of numerical modelling of a structure. But the experimental and numerical results do not agree with each other for various reasons. Hence, model updating is necessary to refine the numerical model of the structure so that it can predict the dynamic characteristics of the actual structure.

The importance and emerging of FEM updating as a subject related to the design, construction & maintenance of mechanical systems and civil engineering structures can be well understood from [5,6, 14]. Now, regarding the various methods of model updating, many methods have been proposed. In most model updating methods difficulty arises in the sense that only limited number of sensors are available while experimenting on the structure and so incomplete mode shapes are obtained and also the modal frequencies obtained may not be necessarily the first few modal frequencies and also the order of modes are altered due to damage in the structure and so mode matching is required while minimizing the measurement error of modal frequencies and mode shape vector. But this difficulty is removed entirely by applying Bayesian probabilistic approach for the model updating procedure, which does not require information about the switching and missing modes. This method of model updating can be well understood in [3, 4, 7, 8, 9, 12, 13] and it proves to be an effective way of model updating specially when incomplete mode shape is obtained experimentally due to limited number of sensors.

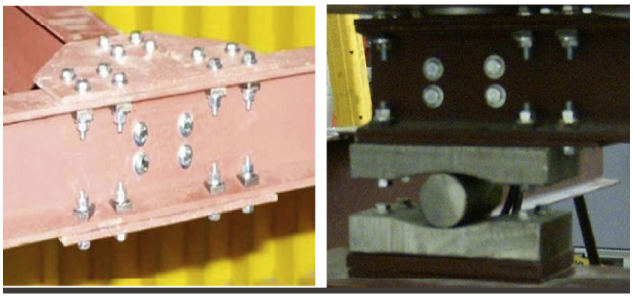
The Bayesian method, which is applied in this paper does not require matching measured modes with corresponding modes from the dynamical model. The method uses an iterative scheme involving a series of coupled linear optimization problems. Also, it does not require solving the eigenvalue problem of the FE model, instead, the eigen equations appear in the prior probability distribution to provide soft constraints. In the present work, the UCF (University of Central Florida) benchmark structure is considered for model updating by applying the Bayesian probabilistic theory and a refined model of the structure is obtained which has dynamic characteristics almost equal to the actual structure.

2. UNIVERSITY OF CENTRAL FLORIDA (UCF) BENCHMARK STRUCTURE

The University of Central Florida (UCF) Grid Benchmark was constructed under the guidance of Catbas et al. [1]. It is a scale bridge model that was designed as a grid system. The UCF benchmark structure serves as a test model for short to medium span highway bridges in terms of the structural response of the structure to ambient vibration. In this paper the model is updated using Bayesian Probabilistic approach considering the test details for the preliminary data for the benchmark structure prepared by Mustafa Gul, PhD student under Dr. F. NecatiCatbas.

Details of the design and construction of the UCF grid are found in Burkett [2] and summarized here. The UCF grid, shown in Fig. 1, has two clear spans, each with a length of 2.74 m (9 ft), and with seven transverse beams connecting the two longitudinal girders. To reduce the effect of support vertical movements, W12x26 sections were used aspiers, each witha length of 1.07 m(42in.) and fixedat the base.While the photo in Fig. 1(a) is a generalphoto showing the experimental setup for the UCF grid, and includes both an

(a)



(b)

Fig. 1: (a) Setup of UCF grid testing (b) UCF benchmark member connections (left) & boundary support (right).

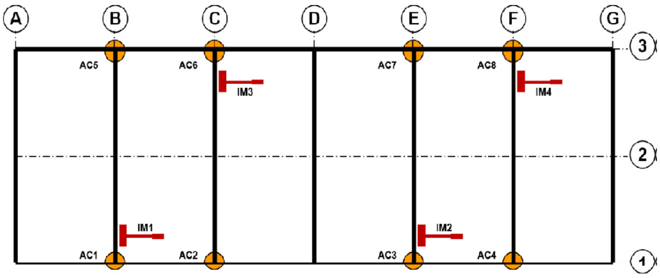


Fig. 2: Accelerometer and impact locations for dynamic testing

actuator and a hammer, only the impact hammer data was used in this paper. As stated in Burkett [2], the transverse beams are connected into girders by two angle clips and two cover plates, using a total of 30,$^1/_4$inch bolts for each connection seen in Fig.1(b).The angle clips provide a shear connection while the cover plates on top and bottom provide a moment. The advantage to this connection is that there are many bolts to loosen and even remove to analyse scenarios such as zero moment transfer and "semi-rigid" connections. The UCF benchmark has complex boundary conditions as shown in Fig. 2. The pier to girder connection was designed to behave as an ideal roller support. It consists of a cylinder sandwiched between two curved bearing plates which allow the translation and rotation in the plane of bending for the girders. Both girder and beams are A36 steel S3x5.7 standard sections.

Eight accelerometers (PCB 393C) were used for dynamic testing on the UCF grid. Each accelerometer was placed vertically and the grid was excited at four different locations. Fig.3 shows the accelerometer and excitation locations for dynamic testing using an impact hammer (IPC 086D20) where AC1, AC2,…, AC8 indicates location of accelerometers and IM1,….,IM4 indicates impacthammer location. Based on the instrumentation of the structure, a multi-input multi-output data set with 4 sets of impact forces and 8 sets of acceleration time histories was created.

A finite element model of the structure was created in SAP2000 as well as the FE dynamic analysis of the structure was done in MATLAB 2013, based on the design calculations and CAD drawings given in Burkett [2]. Fig.3. shows the FE model of the UCF grid structure.The UCF benchmark structure has been modelled using 3d frame elements, link elements etc. with proper support conditions at the base. To have the best a priori model for the purpose of finite element model updating, the authors tried to create the geometry of the UCF grid FE model as accurately as possible. To achieve this goal, the support columns have been extended to the centre of the rotation of roller supports, and the connections between the grid and the support members were modelled using vertical link elements; the six stiffness values of the boundary links can be adjusted to accurately represent the boundary conditions. The UCF benchmark FE model consists of 63 frame elements and 6 link elements. The link elements are used to connect the girder to pier to make it a partially restrained connection. The number of degrees of freedom (DOFs) in this FE model is 316, including single degree offreedom for each link element.

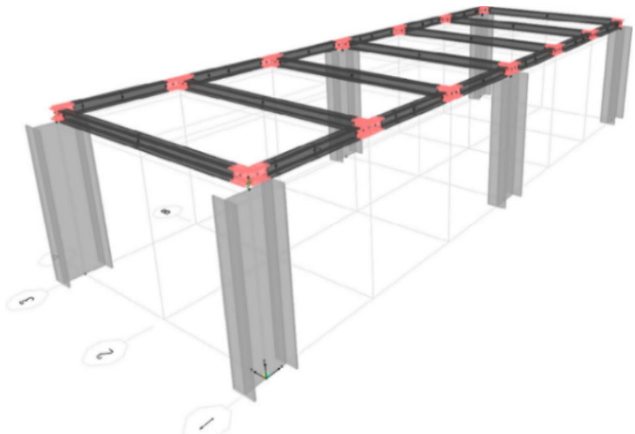


Fig. 3: Finite Element model of the UCF benchmark structure in SAP2000

3. EXTRACTION OF MODAL PARAMETERS

3.1 Experimental Data

The modal data obtained from dynamic test on the test model consists of a set of N_m modal frequencies and N_m incomplete mode shapes. These modal data can be obtained from ambient or forced vibration test data using any reliable modal parameter identification method. In this paper, the modal data consists of Frequency Response Functions (FRFs) for the outputs due to impact test at specified input locations on the structure.

3.2 Complex Mode Indication Function (CMIF) Method

A simple algorithm based on singular value decomposition (SVD) methods applied to multiple reference FRF measurements, identified as the Complex Mode Indication Function (CMIF), was first developed for traditional FRF data in order to identify the proper order of the system equation.
The CMIF is based upon the singular value decomposition of the FRF matrix, containing all possible input –output FRF combinations. The diagonal Singular Matrix ([Σ]) is used as the CMIF. It indicates the existence of real (normal) or complex modes and also gives the relative magnitude of each mode. It is also capable of yielding the corresponding mode shape and/or participation vector. The CMIF plot for the test data obtained from the UCF structure is shown in Fig 4.

4. BAYESIAN PROBABILISTIC APPROACH

4.1. Structural Model Class

The structural model class C, is based on N_d degrees of freedom (DOFs) linear structural models parameterized by the model parameters θ. Considering linear behaviour of the structural dynamic for the identification purposes, a convenient parameterization for the stiffness matrix is-

$$K(\theta) = K_O + \sum_{i=1}^{N_\theta} \theta_i K_i \qquad (1)$$

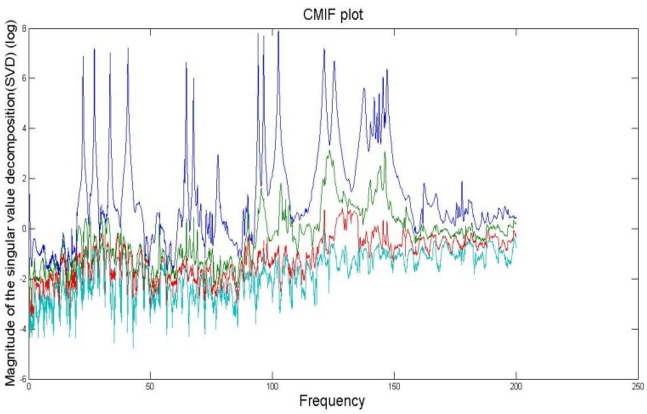


Fig. 4: Complex Mode Indicator Function (CMIF) plot

where,

K_i= the constant matrix independent of θ which can be obtained from FE-model of the structure.

N_θ = number of stiffness parameters.

K_0 = the sub-structure contribution to global stiffness matrix from those elements where no parameterization is considered.

$\theta_i$for i=1,2….. N_θ are the stiffness parameters will need to be updated in every iteration based on observed data from test structure.

The mass matrix could be parameterized using a similar sub-structuring approach, but the writers assume here that it is known with sufficient accuracy from structural drawings so that θ denotes only stiffness-related parameters.

4.2. Formulation of Bayesian Probability Equation

Let '*D*' denotes the observed data from the tested structure, '*θ*' are the model parameters to be updated and as stated above '*C*' is the structural model class. From Bayesian statistics, it is evident that known and unknown parameters are needed. Here, '*D*' is the known quantity and '*θ*' is the unknown quantity. Posterior probability of an unknown quantity is the probability distribution of the quantity, treated as a random variable, conditional on the evidence obtained from an experiment or survey.

Hence posterior or updated probability of the unknown model parameters can be expressed by Bayes' theorem:

$$p(\theta \mid D, C) = \frac{p(\theta \mid D, C)\, p(\theta \mid C)}{p(D \mid C)} \qquad (2)$$

where,

$p(\theta|D,C)$ = the posterior probability density function (PDF) of the model parameters given the observed modal data D and model class C.

$p(\theta|C)$ = the prior PDF of the model parameters based on engineering and modelling judgment.

$p(D|\theta,C)$ = likelihood function.

$p(D|C)$= normalizing constant.

4.3 Formulation of Prior-probability Distribution Function

In Bayesian statistical inference, a prior probability distribution is the probability distribution that would explain one's beliefs about this quantity before some evidence is taken into account.

In this paper, the prior probability density function (PDF) for experimental eigenvalues $\lambda = [\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(Nm)}]^T$ and system eigen vectors $\varnothing = [\varnothing^{(1)T}, \varnothing^{(2)T}, \ldots, \varnothing^{(Nm)T}]^T$ is chosen as :

$$p(\lambda, \phi \mid \theta, C) = c_2 \exp\left[-\frac{1}{2} J_g(\lambda, \phi; \theta)\right] \qquad (3)$$

where c_2 is a normalizing constant and the goodness-of-fit function is given by:

$$J_g(\lambda,\phi;\theta) = A^T \sum_{eq}^{-1} A \quad (4)$$

where,

$$A = \begin{bmatrix} (K(\theta)-\lambda^{(1)}M)\phi^{(1)} \\ (K(\theta)-\lambda^{(2)}M)\phi^{(2)} \\ \vdots \\ (K(\theta)-\lambda^{(N_m)}M)\phi^{(N_m)} \end{bmatrix}$$

Here, the prior covariance matrix Σ_{eq} controls the size of the equation errors, based upon which the prior probability density function (PDF) is formulated. The uncertainty in the equation errors for each mode are modelled as independent and identically distributed, so:

$$\sum_{eq} = \sigma_{eq}^2 I_{N_d N_m} \quad (5)$$

where $I_{N_d N_m}$ denotes the $N_dN_m x\ N_dN_m$ identity matrix and σ_{eq}^2 is a prescribed equation-error variance, which allows for explicit treatment of modelling error as the parametric models for the stiffness matrix, and hence the eigen-equation, is never exact in practice.

The prior PDF $p(\lambda,\phi|\theta,C)$ implies that, given a class of dynamical models and before usingthe dynamic test data, the most probable values of λand φare those that minimize the Euclidean norm (2-norm) of the error in the eigen equation for the dynamical model. This implies that the prior most probable values of λand **φ**are the squared modal frequencies and mode shapes of a dynamical model, but these values are never explicitly required. This prior PDF will have multiple peaks because there is no implied ordering of the modes here. The prior PDF for all the unknown parameters is given by:

$$p(\lambda,\phi,\theta|C) = p(\lambda,\phi|\theta,C)\,p(\theta|C) \quad (6)$$

where the prior PDF $p(\theta|C)$ can be taken as a Gaussian distribution with mean θ^η representing the nominal values of the model parameters and with covariance matrix Σ_θ.

4.4 Formulation of Likelihood-probability Distribution Function

Likelihood function is a function of the parameters of a statistical model. In informal contexts, "likelihood" is often used as a synonym for "probability" .But in statistical usage. A distinction is made depending on the roles of the outcome or parameter. Probability is used when describing a function of the outcome given a fixed parameter value. Likelihood is used when describing a function of a parameter given an outcome. The likelihood of a set of parameter values, θ , given outcome x is equal to the probability of these observed outcomes given those parameter values, i.e.

$$L(\theta/x) = P(x/\theta)$$

In this paper, to construct the likelihood function, the measurement error εis introduced:

$$\begin{bmatrix}\hat{\lambda}\\ \hat{\varphi}\end{bmatrix} = \begin{bmatrix}\lambda \\ L_o ø\end{bmatrix} + \varepsilon \quad (7)$$

Here, εis the difference between measured eigenvalues, mode shapes and analytical eigenvalues, mode shapes. A Gaussian probability model is chosen for $\varepsilon \in R^{N_m(N_o+1)}$ with zero mean and covariance matrix Σ_ε , which can be obtained by Bayesian modal identification methods

$$\hat{\Psi} = \left[\hat{\Psi}^{(1)T}, \hat{\Psi}^{(2)T}, \ldots\ldots\ldots., \hat{\Psi}^{(N_m)T}\right]^T \text{ and}$$

$$\hat{\lambda} = \left[\hat{\lambda}^{(1)}, \hat{\lambda}^{(2)}, \ldots\ldots\ldots., \hat{\lambda}^{(N_m)}\right]^T$$

N_m = Number of measured eigenvalues.

N_o = Number of observed degrees of freedom.

$\hat{\Psi}^{(m)} \in R^{N_o}$ gives the observed components of the system eigen-vector of the m^{th} mode .

$\hat{\lambda}^{(m)}$ gives the corresponding observed system eigenvalue from dynamic test data.

$L_o = N_mN_o x\ N_mN_d$ observation matrix of '1s' or '0s' that picks the components of $ø$ corresponding to the N_o measured DOFs.

The likelihood function is therefore,

$$p\left(\hat{\lambda},\hat{\psi}|\lambda,\phi,\theta,C\right) = p\left(\hat{\lambda},\hat{\psi}\,|\,\lambda,\phi\right) = G\left(\left[\hat{\lambda}^T, \hat{\psi}^T\right]^T; \left[\lambda^T, (\Gamma\phi)^T\right]^T, \sum_\epsilon\right) \quad (8)$$

which is a Gaussian distribution with mean $[\lambda^T, (L_0ø)^T]^T$ and covariance matrix Σ_ε .

4.5 Formulation of Posterior-probability Distribution Function

In Bayesian probabilistic statistics, posterior probability is the conditional probability that is assigned after the relevant evidence or background is taken into account. In this paper, the posterior PDF for the unknown parameters is given by the Bayes' theorem:

$$\begin{aligned} p\left(\lambda,\phi,\theta\,|\,\hat{\lambda},\hat{\psi},C\right) &= k_1 p\left(\hat{\lambda},\hat{\psi}\,|\,\lambda,\phi,\theta,C\right) p(\lambda,\phi\,|\,\theta,C)\, p(\theta\,|\,C) \\ &= k_1 p\left(\hat{\lambda},\hat{\psi}\,|\,\lambda,\phi\right) p(\lambda,\phi\,|\,\theta,C)\, p(\theta\,|\,C) \end{aligned} \quad (9)$$

The most probable values of the unknown parameters can be found by maximizing this PDF. The objective function is obtained by taking the negative logarithm of the posterior PDF without including the constant that does not depend on the uncertain parameters. Hence the objective function is defined as:

$$J(\lambda,\phi,\theta)=\frac{1}{2}(\theta-\theta^{\eta})^{T}\Sigma_{\theta}^{-1}(\theta-\theta^{\eta})+\frac{1}{2\sigma_{eq}^{2}}\sum_{m=1}^{Nm}\left\|(K(\theta)-\lambda^{(m)}M)\phi^{(m)}\right\|^{2}$$
$$+\frac{1}{2}\begin{bmatrix}\hat{\lambda}-\lambda\\ \hat{\Psi}-L_{O}\Phi\end{bmatrix}^{T}\Sigma_{\epsilon}^{-1}\begin{bmatrix}\hat{\lambda}-\lambda\\ \hat{\Psi}-L_{O}\Phi\end{bmatrix} \quad (10)$$

Here, $\|.\|$ denotes the Euclidean norm. Hence the objective function is minimized instead of maximizing the posterior PDF.

4.6 Linear Optimization Formulation

Since sensors in the test model are placed only at specified locations of grid structure, the mode shapes are measured with incomplete mode shape components but the mode shapes are measured with high accuracy. Therefore, the sequence of optimization starts from computing the missing components of the mode shapes. The entire process of linear optimization of the objective function is well explained in [13].

Iterative algorithm

The iterative procedure consists of the following steps:

1. First set the nominal values of the updatedparameters $\theta^{*}=\theta^{\eta}$ and the eigenvalues as the measured values $\lambda^{*}=\hat{\lambda}$.The updated stiffness matrix is $K^{*} = K(\theta^{*})$.
2. Update the estimates of the system eigenvectors $\phi^{(m)*}$, m=1,2,…..,$N_m$by minimizing the objective function in equation (10) with respect to ϕ so that the optimal vector ϕ^{*} is then obtained.
3. Update the estimates of the system eigenvalues $\lambda^{(m)*}$,m=1,2,….,$N_m$by minimizing the objective function in equation (10) with respect to λ so that the optimal vector $\lambda^{*}$is then obtained.
4. Update the estimates of the model parameters $\theta$by minimizing the objective function in equation (10) with respect to θ so that the optimal vector θ^{*} is then obtained.
5. Iterate the previous steps 2,3 and 4 until the model parameters in θ^{*}satisfy some convergence criterion, thereby giving the most probable values of the model parameters based on the modal data.

5. UPDATING OF THE UCF STRUCTURE

5.1 Parameterization of Stiffness matrix

For each frame element, four stiffness parameters for each frame element are considered for parameterization of the stiffness matrix, making a total of 252 stiffness parameters $\theta_{4(l-1)+1}$.= $K_{l,ax}$, $\theta_{4(l-1)+2}$= $K_{l,bz}$, $\theta_{4(l-1)+3}$ =$K_{l,by}$, $\theta_{4(l-1)+4}$.= $K_{l,tr}$, l=1,2,….., 63 where the index l represents the frame number and '*ax*', '*bz*', '*by*', '*tr*' represent the axial component, the bending componentabout z-axis, , the bending componentabout y-axis and the torsional component of the element stiffness matrix respectively. Also one parameter each is considered for 6 link elements, giving a total of 258 stiffness parameters.

Four stiffness parameters considered for each frame element are as follows-

$$\theta_{1,l}=\frac{A_l E_l}{L_l},\theta_{2,l}=\frac{E_l I_{z,l}}{L_l^3},\ \theta_{3,l}=\frac{E_l I_{y,l}}{L_l^3},\ \theta_{4,l}=\frac{G_l J_{,l}}{L_l}$$

5.2 Model Updating Results

The experimental data viz. the natural frequencies and mode shape vectors are obtained as given earlier. For the model updating procedure five initial modes of the structure subjected to vibration are considered. The experimental modal frequencies of the structure are 22.37, 27.01, 33.38, 40.91, 64.93 Hz and partial mode shapes of 8 components.The identified modal frequencies are shown in second column of Table 2.and are referred to as target frequency.

Finite Element Model (FEM) updating of the structure is done using the proposed method mentioned earlier and an optimal model class has been identified based on a large evidence with 1% coefficient of variations of the measurement error of the squared modal frequencies and mode shapes for all modes. The nominal values of the element stiffness parameters are selected from a uniform distribution over $2\tilde{\theta}$to $3\tilde{\theta}$, where $\tilde{\theta}$ is the actual stiffness parameter. The Table2. shows the initial target and identified values of the modal frequencies, modal assurance criteria (MAC) values and correlation coefficients of corresponding modes before and after updating. It is clearly seen that the updated modal frequencies are very close to the target values. Also MAC values and correlation coefficient values are improved due to the updating procedure.

In Fig 5., the iteration histories of few stiffness parameters are shown and in Fig 6., the comparison in the correlation matrix between experimental and model shapes before and after updating are shown and it is clearly seen that the correlation coefficients and MAC values get improved due to model updating which indicates that the updated mode shapes get close to experimental modes. Also, the updated values of few of the representative parameters are shown in Table 1.

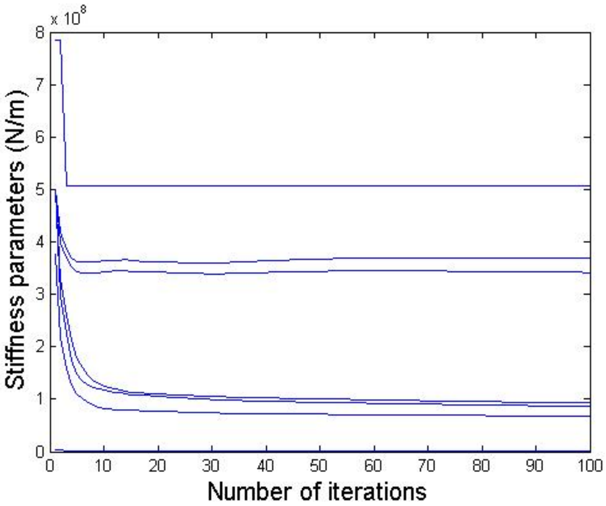


Fig. 5: Iteration histories of few representative stiffness parameters

Table 1: Updated values of few representative stiffness parameters

Parameter Sl. No.	Stiffness parameter value(before updating) (kN/m)	Stiffness parameter value (after updating) (kN/m)
1	1921052.63	4697645.26
2	1921052.63	4802308.71
3	1921052.63	4339442.37
4	1921052.63	4724461.30
5	313811.18	784527.97
6	313811.18	456627.48
7	313811.18	774761.39
8	183997.23	50976.39
9	648.74	159.02
10	150122.69	366641.07

Table: 2 Results of updated modal frequencies, MAC values and Correlation Coefficients

Experimental modes	Target values	Initial Values	Updated values	MAC values		Correlation coefficient	
Order of modes	f(Hz)	f(Hz)	f(Hz)	Initial	Updated	Initial	Updated
1.Vertical Bending	22.31	23.13	22.22	.1153	.9999	-.3592	.99
2.Vertical Torsion	26.93	27.77	26.34	.1259	.9997	-.3852	.99
3.Vertical Bending	33.375	38.08	32.83	.0144	.9750	-.0633	.97
4.Vertical Torsion	40.75	52.77	39.74	.0812	.9984	.3064	.99
5.Vertical Bending	64.68	54.52	65.80	.0250	.9995	.1571	.99

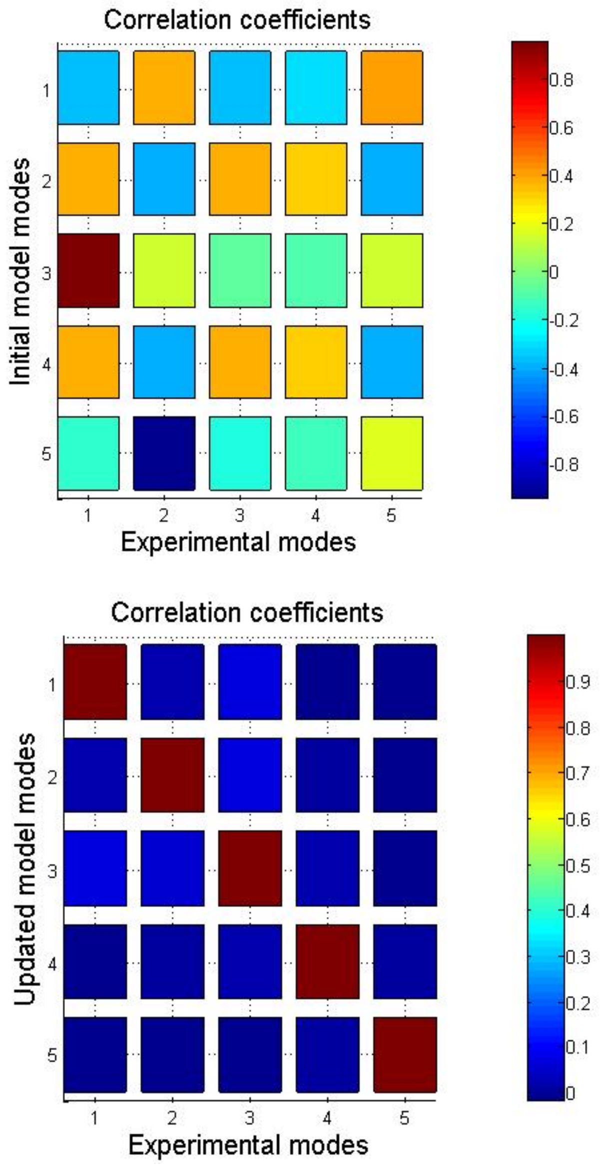


Fig. 6: Comparison of correlation matrix profile of experimental and model modes before and after updating

6. CONCLUSION

Dynamic testing of a structure results in incomplete modal data and also the result is subjected to errors due to noise in measurement procedures. Also FE modelling of the structure is subjected to modelling errors. Hence, actual representation of the structure is not possible. But by using Bayesian probabilistic model updating, the uncertainties in measurement and modelling errors are limited to find a baseline (undamaged) structure. The model parameters are updated following an iterative procedure involving linear optimization so that the most probable parameters are obtained, such that frequencies and mode shapes of the updated model is in close correlation with that obtained from experimental results. The advantage of this method is that the incomplete mode shape data obtained experimentally does not cause any problem and the updated model can be efficiently obtained.

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